Design of an Interdigital Hairpin Bandpass Filter Utilizing a Model of Coupled Slots

Anatoli N. Deleniv, Marina S. Gashinova, Irina B. Vendik, *Member, IEEE*, and Anders Eriksson, *Member, IEEE*

***Abstract*—A design methodology of an interdigital hairpin filter based on a full-wave model of coupled slots is developed and discussed. It is applied to design a five-pole bandpass filter centered at 1.7 GHz with 1.75% relative bandwidth. A low level of in-band insertion loss is achieved using high-temperature superconductor electrodes. Experimental data are presented and compared with the simulated response. A model of coupled slots embedded in a layered lossy media is developed on the basis of the spectral-domain approach. The multiport admittance (impedance) matrix of the coupled slots is formulated using the E–H duality principle. It is successfully applied in the design of the filter.**

***Index Terms*—Coplanar waveguides, duality principle, microwave filters, spectral-domain approach.**

I. Introduction

HIGH-QUALITY microwave filters play an extremely important role in designing communication systems, such as cellular communication and global-range navigation systems. For some applications, the requirement of low insertion loss is crucial. In these cases, the use of high-temperature superconductors (HTSs) is very attractive since it allows the design of highly selective filters. Numerous research groups have put large efforts to design such devices based on microstrip lines [1]–[8], whereas only a limited number of publications deals with coplanar waveguide (CPW) filter structures. This is particularly due to the absence of accurate models for computer-aided design (CAD) and the necessity to use wire bonding frequently. On the other hand, coplanar technology is advantageous of being uniplanar and offering greater flexibility in miniaturization.

A number of practically useful CPW filter topologies have been presented in the literature [9]–[15] using either CPW discontinuities or edge-coupled CPW sections. It is important to notice that it is often necessary to characterize the discontinuities experimentally [9], [11], resulting in a lengthy and less effective filter design. Most of the referred filters have a moderate-to-wide bandwidth, except the 1.8% HTS bandpass filter reported in [11].

In this paper, we present the design, fabrication, and experimental characterization of a recently introduced CPW-based filter with an interdigital structure [16]. The topology utilized provides a more compact filter compared to other types and it can be designed quite accurately. Also presented in this paper is an effective CAD-oriented model of coupled slots for circuit simulations developed to support the filter design. It enables accurate computation of complex propagation constants and associated modal powers on the basis of the spectral-domain approach (SDA). The multiport admittance matrix for an array of coupled slots is formulated using the E–H duality principle [17]. It is successfully applied in the design of the filter that utilizes hairpin slot-line resonators. In combination with recently developed CAD models for CPW discontinuities [18], [19], this model of coupled slots establishes a powerful basis for the design of coplanar (slot-like)-based structures.

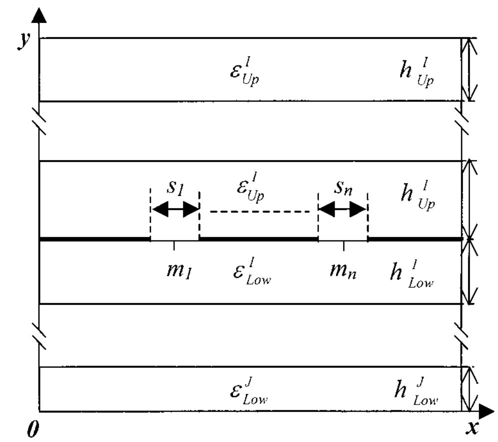

Fig. 1. Generalized coupled-slot structure.

II. Model of Coupled Slots Based on the Hybrid-Mode Analysis

The model is developed in three steps and is described as follows.

Step 1) In the first step, one defines the complex wave propagation vector $k_z(\omega) = \beta(\omega) - j\alpha(\omega)$ in the wave propagation direction z (β being the phase constant and α the attenuation constant) for the structure shown in Fig. 1. The structure consists of $i = 1, \ldots, I$ upper and $j = 1, \ldots, J$ lower dielectric layers with respect to the electrode interface with $k = 1, \ldots, n$ slots (we assume that electrodes are made of ideal metal). We use an analytical procedure for evaluation of the spectral dyad based on the so-called immitance approach [20]. The recursive use of the formula for the input admittance of loaded line section for both TE-to-y

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and TM-to-y waves enables to build flexible codes for an arbitrary number of upper and lower layers related to the electrode interface. An extension to the lossy case is made by use of the complex permittivity $\varepsilon^{i(j)}$ for ith (jth) layer and the complex propagation constant. Following spectral-domain moment method, it is necessary to expand the field in the slot region in an N-dimensional basis set. In our code, a single set is used to model the transverse component of electric field E_x and the derivative of the longitudinal E_z' as follows:

$$E_x(x) = \sum_k \sum_p a_p^k \varphi_p^k(x) \tag{1a}$$

$$E_z'(x) = \sum_k \sum_p b_p^k \varphi_p^k(x) \tag{1b}$$

with

$$\varphi_p^k(x) = \begin{cases} \dfrac{2}{\pi s_k} \dfrac{T_p\left(\dfrac{x-m_k}{0.5 s_k}\right)}{\sqrt{1-\left(\dfrac{x-m_k}{0.5 s_k}\right)^2}}, & \text{for } x \in s_k \\ 0, & \text{for } x \notin s_k \end{cases} \tag{2}$$

where $k = 1, \ldots, n$ stands for the kth slot and p stands for the order of the basis function. Our choice of this basis set is based on the fact that it provides description of the edge singularity problem and is also due to the possibility of using an effective speed-up technique [21]. Now the following equality holds in the spectral domain:

$$I_z(x) = \int_{-L}^{x} J_z(x')\, dx'$$

$$\begin{bmatrix} \tilde{J}_x \\ \tilde{I}_z \end{bmatrix} = \begin{bmatrix} \tilde{Y}_{xx} & \dfrac{\tilde{Y}_{xz}}{-j\alpha_n} \\ \dfrac{\tilde{Y}_{zx}}{-j\alpha_n} & \dfrac{\tilde{Y}_{zz}}{-(\alpha_n)^2} \end{bmatrix} \begin{bmatrix} \tilde{E}_x \\ \tilde{E}_z' \end{bmatrix}. \tag{3}$$

Solving (3) by Galerkin's method leads to a homogeneous system of linear equations

$$[A(\omega, k_z)]\,[X] = 0 \tag{4}$$

with a certain coefficient matrix $[A(\omega, k_z)]$ and $[X] \equiv a \cup b$ being unknown expansion coefficients in (1a) and (1b). Now the computation routine based on Newton's method is applied to find a set of complex roots of (4). As a final result of this step, we obtain complex wavenumbers k_z^k, $k = 1, \ldots, n$ of propagating modes.

Step 2) The goal of this step is the calculation of powers associated with each particular mode. We use [22, expr. 9] for computation of the power flowing in the ith layer in terms of the tangential electric field at the surfaces limiting this layer. In order to provide a connection between the electric field at the electrode interface and any other interface, a set of simple formulas was derived within the immitance approach and used in our routine. As a result of this step, we obtain an $n \times n$ diagonal matrix $[P]$, in which the elements are modal powers.

Step 3) The goal of this step is to define a multiport admittance (impedance) matrix in order to perform circuit simulations. The most natural way is to formulate this problem regarding equivalent magnetic currents. For n slots placed in the electrode interface, we have $n-1$ quasi-TEM modes and one quasi-TE finline associated mode with a certain cutoff frequency. Adjusting the transverse box dimensions, we can easily control the existence of this quasi-TE mode, thus, an $n \times n$ multiport admittance matrix can be formulated as follows. First, we compose magnetic current eigenvectors $[Im_k]$ for all $k = 1, \ldots, n$ modes. The entries of each eigenvector are calculated using the following expression:

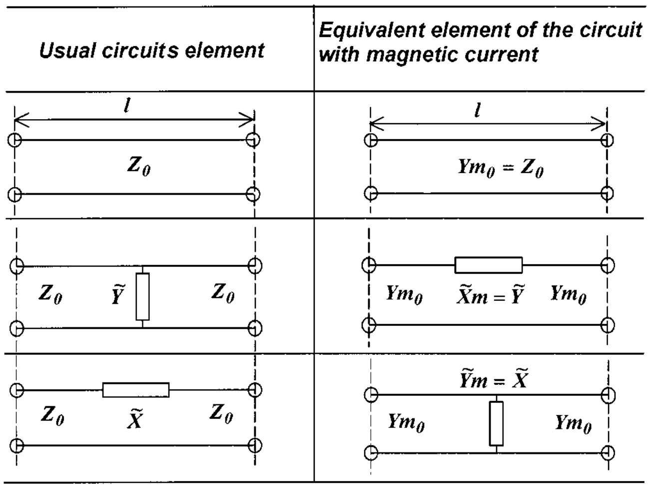
$$Im_{i,k} = \int_{m_i - 0.5 s_i}^{m_i + 0.5 s_i} Jm_z(x) dx \tag{5}$$

where $\tilde{J}m_z \hat{z} = \tilde{E}_x \hat{x} \times \hat{n}$ ($\hat{n}$- is a unit vector normal to the electrode interface). In other words, the longitudinal magnetic current is represented by a complex coefficient related to the first term of the field expansion $\tilde{E}_x$ on each slot since only this term contributes to the integral with this choice (2) of basis functions. Now, following a well-established procedure, we can compute associated voltage eigenvectors $[Vm_k]$ by solving a set of linear equations as follows:

$$\left.\begin{aligned} P^k &= \frac{1}{2} Vm_k^T Im_k^* \\ 0 &= Vm_l^T Im_{ml \neq m} \end{aligned}\right\}, \qquad k, l, m = 1, \ldots, n. \tag{6}$$

Modal admittances used to define multiport admittance matrices of the coupled-line system [23] are defined for each lth line and each kth mode as a ratio between related elements of matrices $[Im]$ and $[Vm]$ in such a way that $Ym_{l,k} = (Im_{l,k})/(Vm_{l,k})$. It will be shown later that this formulation establishes a powerful basis for circuit simulations and is connected with some simple rules of replacement between elements of a usual circuit and one with a magnetic current. The order of replacement between circuits related to a single slot line with either electric or magnetic currents is summarized in Table I and is obtained utilizing a modulus equality of related reflection coefficients. Both circuits constructed in accordance to this table have the same modulus of the reflection coefficient with a phase shift π. In order to demonstrate the utility of the proposed formulation, attention is paid to Fig. 2(a), where a piece of a coplanar line between two air bridges (marked as black rectangles) with an inductive stub is sketched. It is not obvious how to define an equivalent circuit with electric current within this topology representation. However, following Table I, we can easily do it

TABLE I
REPLACING ORDER BETWEEN ELEMENTS OF USUAL CIRCUIT AND ONE WITH MAGNETIC CURRENT



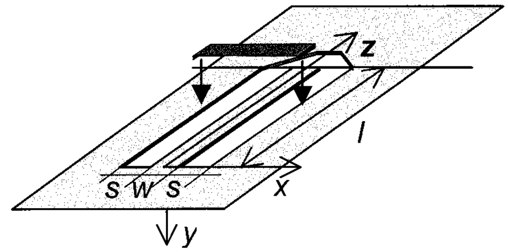
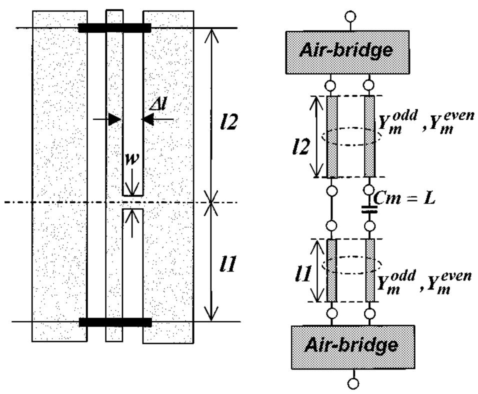


Fig. 4. Slot-line hairpin resonator.



(a) (b)

Fig. 2. (a) Coplanar line with inductive stub. (b) Its equivalent circuit for magnetic current.

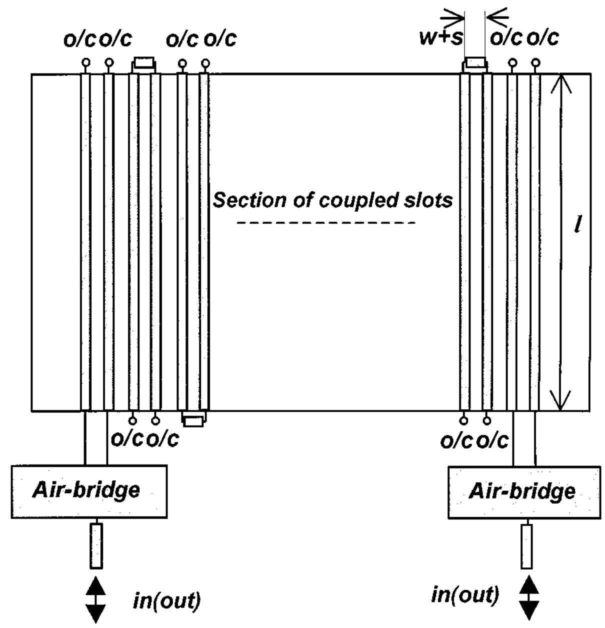


Fig. 5. Equivalent circuit for analysis using the model of coupled slots.

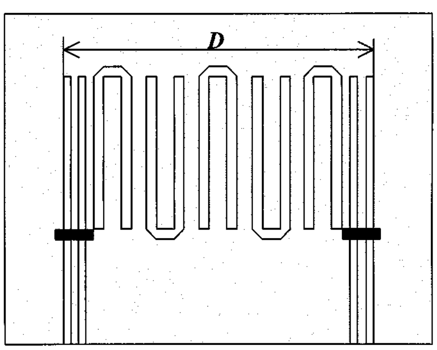


Fig. 3. Five-pole filter based on a slot-line hairpin resonator.

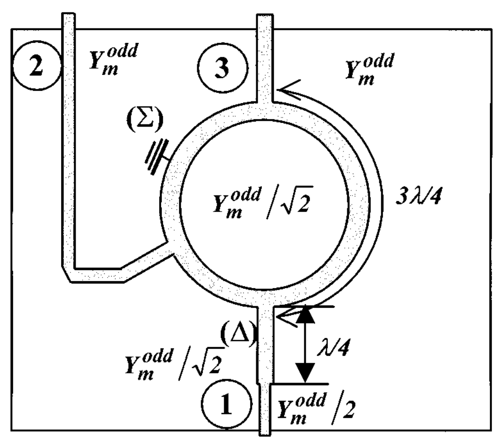
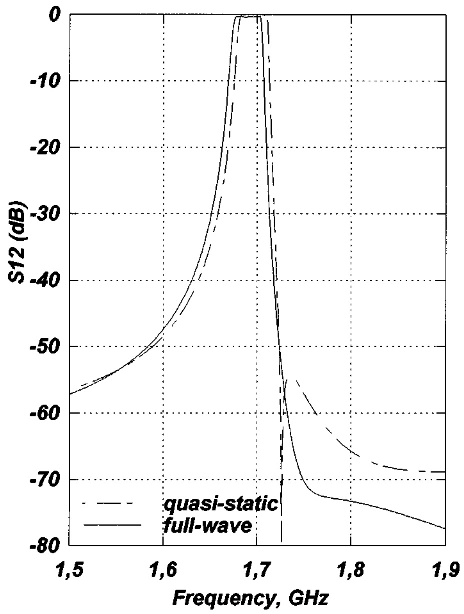


Fig. 6. Equivalent circuit for air bridge.

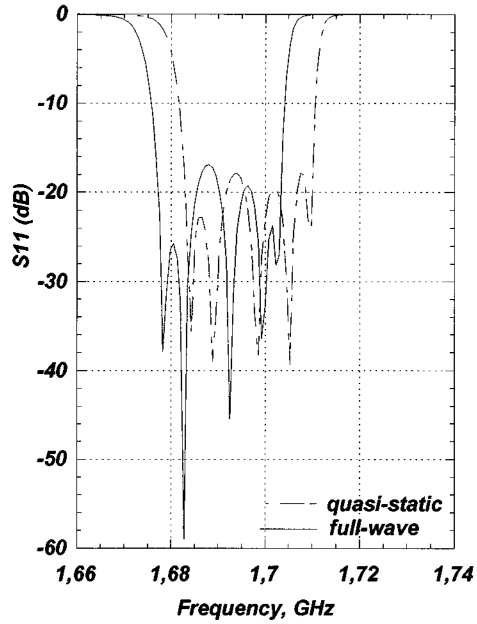
in terms of a circuit with magnetic current depicted in Fig. 2(b). It consists of two coupled-line sections with $l1$ and $l2$ lengths, respectively, separated by an equivalent series capacitance in one of the lines. $Y_m^{\rm odd}$, $Y_m^{\rm even}$ account for both coplanar and slot modes, while the coupling capacitance in the first approach can be simply defined by the value of the stub inductance. The remaining question of the equivalent circuit for air bridges is discussed and resolved in Section III.

III. Filter Design

The layout of the filter to be designed is depicted in Fig. 3. A substrate of a finite thickness is embedded inside a shielding box and is used as a support for the metallized plane of the filter structure. The array of slot-line hairpin resonators is coupled



(a)



(b)

Fig. 7. Comparison of the filter performance obtained using a quasi-static and full-wave approach for the: (a) transmission coefficient and (b) reflection coefficient.

with the external coplanar feed lines using a section of two coupled slots. Two air bridges (marked as black rectangles) are used to isolate the slot mode excited in this section from the source (load). According to the developed design procedure, we follow two steps.

Step 1) An initial topology of the filter is synthesized using the quasi-static approach;

Step 2) The full-wave model of coupled slots is applied to optimize the geometry.

A close-up of the hairpin resonator used in this structure is shown in Fig. 4. It consists of a section with two coupled slots connected at $z = l$ embedded in an infinite metallization plane. For given characteristic dimensions (s, w, l), the lowest resonance frequency belongs to the even mode with a magnetic wall placed in a symmetry plane y-z; one can say it is a coplanar quarter-wavelength resonator short circuited at $z = 0$ and open at $z = l$. In order to provide a ground equalization (i.e., to suppress the odd-mode field with an electric wall in the symmetry plane) a few air bridges have to be placed along the resonator with a certain step—these are omitted in the final design and presented here only to justify the applicability of the quasi-static approach. Now, the field of the constructed resonator can be approximated by a TEM coplanar mode, which enables us to utilize the design procedure developed in [24] for interdigital filters. The original theory is applicable only for a uniform media suggesting that phase velocities of all propagating modes are the same. This criterion holds only for sufficiently thick substrates, while for comparatively thin ones, there is a clear necessity of additional efforts. A detailed design methodology for this case can be found in [18]. Following this methodology, the geometry of a five-pole filter with a 1.75% passband was synthesized on a basis of the resonator with characteristic dimensions $w = 0.25$ mm, $s = 0.2$ mm, and $l = 12.4$ mm. For this case, the characteristic size of the filter is $D = 10.3$ mm (Fig. 3). 1-mm-thick $LaAlO_3$ $(\varepsilon = 23.7)$ coated on one side by 0.5-μm-thick Y–Ba–Cu–O film was used in the design. The effects of the line extension due to open-end discontinuities where taken into account using SDA-based quasi-static model developed in [18]. As stated above, air bridges for the ground equalization within resonators were omitted in the final topology (Fig. 3) or, in other words, the final layout is not equivalent to the used electrical scheme anymore. Thus, the optimization procedure based on the coupled-slot model, developed above, should be applied to account for all excited modes in the structure. The equivalent circuit used for the optimization is presented in Fig. 5. A coupled-slot section, with some ends being open circuited (*o/c*) and some connected by a slot line with characteristic length $w + s$ is fed through the blocks marked as an "air-bridge" by coplanar lines. Let us take a look at Fig. 6, where an equivalent circuit for "air-bridge " is sketched. It is easily seen that it consists of a ring hybrid with a shorted sum port and a differential one connected with the feed line using a matching quarter-wavelength transformer. The feed line represents two coupled slots used as a guide only for the coplanar mode (odd mode in terms of magnetic current); thus, it is characterized by admittance $Y_m = Y_m^{\mathrm{odd}}/2$. This mode traveling from port 1 excites two waves equal in amplitude, but with π phase shift in ports 2 and 3. In this way, the effect of a coplanar feed line connected to two coupled slots is described. Another wave incoming to ports 2 and 3 from this section is a superposition of both (even and odd modes). While for the coplanar mode it is matched at port 1, an even mode in terms of magnetic currents is completely reflected due to the shorted sum port. Using this equivalent scheme, we establish the basis for optimization utilizing the model of coupled slots presented.

As expected, minimal additional efforts where applied to adjust the geometry. Comparative performance of the filter simulated with initial topology using quasi-static and hybrid-mode modeling is presented in Fig. 7. The small frequency shift be-

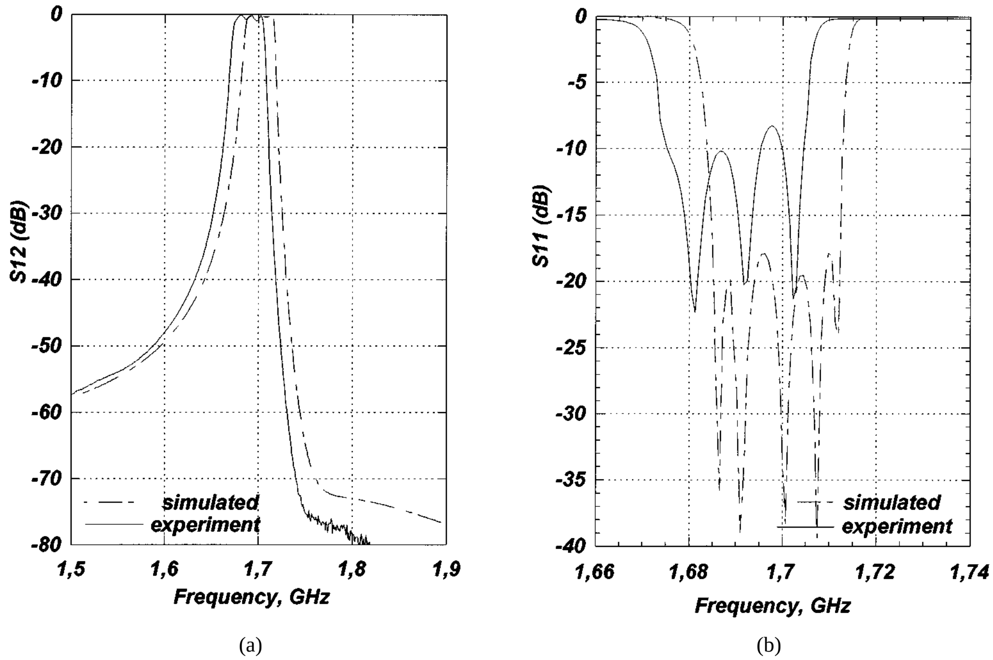


Fig. 8. Comparison of the simulated and experimental filter performance for the: (a) transmission coefficient and (b) reflection coefficient.

tween simulated characteristics is probably due to the effect of dispersion, which is not negligible even at such low frequencies. The most important difference to be clarified experimentally is the presence or absence of a pole on the right-hand-side skirt of the transmission characteristic. A 25×25 mm sample with developed filter was manufactured and measured in the standard Wiltron test fixture at liquid nitrogen temperature (77 K) using an HP 8510 network analyzer. Experimental data are compared with an optimized simulation and presented in Fig. 8. No trimming was used during measurement. We indicate good agreement between simulated and experimental data for the transmission coefficient, while for the reflection coefficient, it is worse. One possible explanation might be the fact that the calibration procedure for a 3.5-mm coaxial line was used; thus, a possible mismatch of coaxial-to-coplanar line transition could affect the performance of the reflection coefficient. An averaged level of insertion losses in the passband was as low as -0.45 dB. A small frequency shift is probably due to a higher dielectric permittivity of the substrate. We suggest that the good coincidence of experimental and simulated data is a reliable confirmation of the hybrid-mode coupled-slot model developed above.

IV. Conclusion

An accurate model of coupled slots based on the SDA is developed. In combination with recently developed models for a number of coplanar discontinuities, it establishes a powerful tool for the design and optimization of coplanar (slot-like) structures. A design methodology for a interdigital hairpin filter is demonstrated in the design of the five-pole bandpass filter with a 1.75% fractional bandwidth. Good agreement between the simulated and experimental performance indicates both correctness of the design procedure and the accuracy of the coupled slot model.

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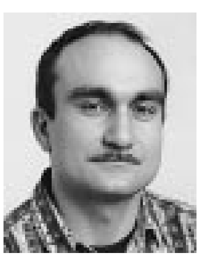

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